110[F].—Ju. V. Linnik, *The Dispersion Method in Binary Additive Problems*, Translations of Mathematical Monographs, Vol. 4, American Mathematical Society, Providence, Rhode Island, 1963, x + 186 pp., 23 cm. Price $12.30.

In recent years the author has had some spectacular successes with problems in the additive theory of numbers that seem to lie just beyond the reach of the now classical Hardy-Littlewood-Vinogradov method. He achieved these remarkable results by combining the methods of analytic number theory with some elementary tools from probability theory, specifically, the concepts of dispersion and covariance and the Chebyshev inequality. The present book is devoted to a systematic exposition of this work. Since the problems involved are old and difficult ones, the detailed proofs require elaborate computations, and are by no means easy to read.

An example of the results obtained in this book is an asymptotic formula for the number $Q(n)$ of solutions of $x^2 + y^2 + p = n$, x and y integers, p prime. Linnik proves that for large n we have

$$Q(n) = \pi A h(n) \frac{n}{\log n} + O\left(\frac{n}{(\log n)^{1.028}}\right),$$

where

$$A = \prod_p \left\{1 + \frac{\chi(p)}{p(p-1)}\right\}, \qquad h(n) = \prod_{p \mid n} \left\{\frac{(p-1)(p-\chi(p))}{p^2 - p + \chi(p)}\right\},$$

the products being taken over the odd primes and $\chi(p)$ being an abbreviation for $(-1)^{(p-1)/2}$. This asymptotic formula, along with many similar assertions, was conjectured by Hardy and Littlewood in their paper, "Some problems of partitio numerorum III: On the expression of a number as a sum of primes," *Acta Math.*, v. 44, 1923, pp. 1–70. A systematic tabulation of the present status of the many interesting conjectures made in this famous paper may be found in a recent note by A. Schinzel, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447.

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111[F].—Albert Leon Whiteman, Editor, *Theory of Numbers*, Proceedings of Symposia in Pure Mathematics, Volume VIII, American Mathematical Society, Providence, R. I., 1965, vii + 214 pp., 26 cm. Price $7.10.

There are given here 22 of the lectures presented at an AMS number theory symposium at the California Institute of Technology on November 21–22, 1963. The lectures will be of much interest to many readers of this journal, especially since a number of them touch upon, or refer directly, to work that has appeared here in recent years. See, for example, the papers of Bateman, Carlitz, Mills, and Cohn. The paper of Bateman and Horn is discussed, at length, in the following review.

An unforgettable episode at the symposium was the disruption and termination of its third session upon receipt of the news that President Kennedy had been killed.

The papers here are by Atle Selberg, R. R. Laxton and D. J. Lewis, W. J. LeVeque, Marshall Hall, Jr., Albert Leon Whiteman, Basil Gordon and E. G. Straus, Kenkichi Iwasawa, Morris Newman, E. C. Dade and O. Taussky, E. T. Parker, Gordon Pall, B. J. Birch, N. C. Ankeny, Paul T. Bateman and Roger A. Horn, Tom M. Apostol, S. Chowla and H. Walum, L. Carlitz, W. H. Mills, Leo Moser, P. Erdös, Harvey Cohn, and J. Lehner.

D. S.

112[F].—Paul T. Bateman & Roger A. Horn, "Primes represented by irreducible polynomials in one variable," *Theory of Numbers* (see previous review), pp. 119–132, in particular Tables II–V.

This paper is concerned with further development of a topic previously examined in this journal in references [1], [2]. The question is to estimate the number of integers n between 1 and N for which $f_i(n)$ $(i = 1, 2, \cdots, k)$ are simultaneously primes, where the f_i are distinct, irreducible polynomials. Under broad conditions, Bateman has conjectured that this number $P(N)$ satisfies

$$P(N) \sim c \frac{N}{(\log N)^k} \tag{1}$$

where the constant c is given by an explicit slowly convergent infinite product.

In a series of papers, [3]–[8], the reviewer had developed techniques of accurately computing these constants c for, say, $k = 1$ and $f_1 = n^4 + 1$ or $f_1 = n^2 + a$, and for $k = 2$ and $f_{1,2} = (n \pm 1)^2 + 1$. Bateman points out here that in all these cases the f_i are *abelian* polynomials, and he gives a general approach to the problem for any abelian polynomials. This general attack, like the specific ones mentioned, uses certain Dirichlet series, but it does not attain the degree of convergence which had been obtained in those special cases.

The authors also examine here (among others) eight *non-abelian* cases: $x^3 \mp 2$, $2x^3 \mp 1$, $x^3 \mp 3$, $3x^3 \mp 1$, and they give empirical counts of such primes for $x <$ 14000, 6000, 14000, and 8000, respectively. But for these non-abelian cases *no accurate way of computing the constants is known.* For example, the number of primes of either form $n^3 \mp 2$ is conjectured to satisfy

$$P(N) \sim \frac{1}{3} A \int_2^N \frac{dn}{\log n} \tag{2}$$

where

$$A = \prod_p \frac{p - \alpha(p)}{p - 1}, \tag{3}$$

the product being taken over all primes $p = 6m + 1$ with $\alpha(p) = 3$, or 0, according as p is, or is not, expressible as $a^2 + 27b^2$. The sequence of partial products here not only converges very slowly, but has an annoying, irregular "drifting" character that frustrates any standard acceleration technique. In the limit, there are twice as many primes p with $\alpha(p) = 0$ as with $\alpha(p) = 3$, (that is why the product converges), but the two types of p occur in a "random" manner, and this causes the sequence to drift up and down in a way that defies the instinct of any numerical analyst. Presumably, a Césaro sum would help some, but that is not very satis-